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Use of Dirac like matrices to compute the wave propagation in various medium

O.Maurice¹, A.Reineix²

¹GERAC ZAC Pissaloup 3 rue Jean d'Alembert 78190 Trappes, France.
olivier.maurice@gerac.com

²Xlim 123 avenue Albert Thomas 87060 Limoges, France. alain.reineix@xlim.fr

1 Introduction

One problem in electromagnetic compatibility for embedded systems is to evaluate the energy exchange of one electronic with its near environment. These interactions explains the self disturbances between equipemnt enclosed in the embedded system. In the conception phases of the project, there is no time to launch complete simulation and anyway, it would no give the suited results as the Maxwell's software only give the field distribution and don't make the link with the electronic. In order to have at least one response to this problem regarding the far field interactions between elements, we have developed a new technique that can be used for various kind of waves: fields or power density, and perhaps others again such as thermal. The purpose of this paper is to present this technique.

2 Definitions of the objects used

The problem is defined through junctions, tubes, sources and loads. The tubes are connected on the junctions through ports. The sources and loads are connected too on junctions with their own ports. Figure 1 gives an example of some topology with three junctions, three tubes and eight ports.

Starting from this topology we define two matrices, two tensors and two vectors. The first matrix is the distribution matrix giving the relations there is between ports on each junctions. This matrix does not depends on time. For the case quoted above, the S matrix (related in fact with the classical one in high frequency engineering, but its coefficients can have different significations) has the form:

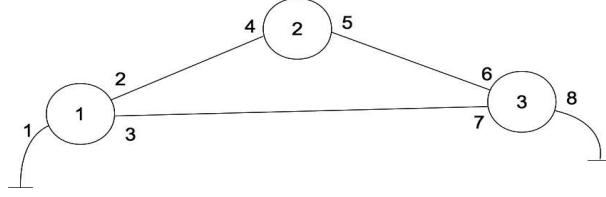


Figure 1: Figure 1

$$S = \begin{bmatrix} S_1^1 & S_2^1 & S_3^1 & 0 & 0 & 0 & 0 & 0 \\ S_1^2 & S_2^2 & S_3^2 & 0 & 0 & 0 & 0 & 0 \\ S_1^3 & S_2^3 & S_3^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_4^4 & S_5^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_4^5 & S_5^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_6^6 & S_7^6 & S_8^6 \\ 0 & 0 & 0 & 0 & 0 & S_6^7 & S_7^7 & S_8^7 \\ 0 & 0 & 0 & 0 & 0 & S_6^8 & S_7^8 & S_8^8 \end{bmatrix} \quad (1)$$

This matrix can be constructed knowing that, if S_N is the S matrix of the junction N , the global S-matrix of the system studied is a purely diagonal one and can be synthesized in:

$$S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} \quad (2)$$

The S_k submatrices are not homogeneous in dimensions, as they are linked with the junctions number of connexions with ports. In the previous expression, 0 are also zero matrices of various dimensions. The second matrix is the propagation matrix, responsible of the wave displacement between the ports of the junctions through the tubes. We will discuss later of its expressions. At the opposite to S , this matrix we call G is purely non diagonal. For our case, it has the form:

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_7^3 & 0 \\ 0 & G_2^4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_5^6 & 0 & 0 & 0 \\ 0 & 0 & G_3^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

It's difficult to synthetize the G -matrix similarly to the S one, i.e. that visually, the correspondance is not so evident. But its organization follows the next form if G_m^n is the link between the junctions n and m :

$$G = \begin{bmatrix} 0 & G_1^2 & G_1^3 \\ G_2^1 & 0 & G_2^3 \\ G_3^1 & G_3^2 & 0 \end{bmatrix} \quad (4)$$

To compute the waves propagation, we have to define the sources. These sources form a contravariant vector, under the tensorial analysis of networks [1, 2], where each element refers to a source connected to a port of a junction. If only one source exists in our problem, the vector we call p has the expression (we use the tensorial notation as defined in [1, 3]):

$$p^a = \begin{bmatrix} p^1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

As previously, the various matrices are not homogeneous in dimensions. Between the source, generally coming from a definition in power or energy, so in volt, ampere, watt or joule, and the waves, there is an operator that transforms this power in field, power density, etc. This operator is a metric, twice covariant tensor $g_{\mu a}$. The source p^a has the dimension of a flux (current, impulsion, ...) and the result we will define later is an effort (electromotive force, ...). For example, if we consider the metric g making the transformation from an element of current to the potential vector waves, it has the dimension of $[m]^{-1}[H]$. It appears that, normalizing the inductance value, this dimension is directly linked to the inverse of a distance between the element of current and the field. A similar object exists in the reception stage. A port is linked with a load (it's possible to have many ports of sources and loads). The transformation of the field (in the previous case) into an available voltage on the load (an effort) is realized through a second metric, $h_{b\nu}$. This second object that could be dimensioned in $[m]$. Both g and h are constructed from the identity matrix of dimension $N.N$ if N is the number of ports, changing the port diagonal elements of sources q and loads r where we want to compute the transformations to g_{qq} and h_{rr} . We will see that following a given process we can compute automatically

the wave intensity at each port of the network using the previous matrix and tensors. We will further detail how to extract the rigorous expressions of the waves from the processes applied. Note that, in the particular case of antenna problems, g is the generalized gain of the emitters and h the effective section in reception mode.

3 General process to obtain the waves and energy expressions

Always for illustrating the technique with the network presented figure 1. We accept the idea that the source component p^1 is a temporal wave packet of the form:

$$p^1(t) = p_0 e^{-\left(\frac{t-\tau_0}{\sigma}\right)^n} \sin(\omega_0 t) \quad (6)$$

τ_0 being a delay and σ the pulse typical duration. The exponent n determines the wave packet profile. ω_0 is the wave packet principle pulsation. The main characteristic of the technique is that there is no correlation between the tube characteristics (impedance, delay, ...) and the time propagation. By this way, the wave packet duration is not in relation with the tube characterization. The process is the following: to illustrate it we imagine a source on port 1 and a load on port 8. At the beginning of the process, a source supplies the network. It means that a process $g_{11}p^1$ is applied on port 1. This pulse is transmitted through the junction 1 to the ports 2 and 3. This is translated by the operation: $S_1^2 g_{11}p^1$ and $S_1^3 g_{11}p^1$. After, these impulsions are transmitted through the tubes 2-4 and 3-7 to the ports 4 and 7, thanks to the $G_2^4 S_1^2 g_{11}p^1$ and $G_3^7 S_1^3 g_{11}p^1$ operators (G_2^4 and G_3^7 being operators that we will detail further). Imagine that we want to extract the current wave state to the port 8. We just have to use the metric h_{88} and to distribute the wave at the port 7 to the port 8. It means first to apply the distribution S_7^8 and secondly to transform it using h_{88} . Finally the complete process at this step is given by: $h_{88} S_7^8 G_3^7 S_1^3 g_{11}p^1$. This can be synthesized writing: $\gamma_1^7 = G_3^7 S_1^3$. The previous expressions are given computing $V_\alpha = G_\alpha^\nu S_\nu^\mu g_{\mu\alpha} p^a$. V_α is the wave vector giving the wave states to all the ports of the network (wave ports, not the loads and source ports). Using the γ -matrix, V_α becomes equal to $\gamma_\alpha^\mu g_{\mu\alpha} p^a$. In general, the wave vector will be obtained through products of the γ matrix like: $V_\xi = \gamma_\xi^\beta \gamma_\beta^\zeta \gamma_\zeta^\mu g_{\mu\alpha} p^a$ for three tubes traveled. To obtain the wave energy w_i at one particular port, we make: $w_i = h_{i\beta} S_\alpha^\beta V_\alpha$. Now, if we want to compute all the ports states at the next step, we just have to apply one more the propagation product $G_\alpha^\nu S_\nu^\mu$. It appears that the wave vector components change following the propagation of the waves between all the ports, each time the propagation product is applied. But, we will see

that the link with the space time is not directly dependant with the step of computation. Nevertheless, the wave process is guided through the successive application of the propagation product. For example, applying twice the product, we find on the port 2, i.e. as second component of the wave vector V_α , the reflexion to the source port given by: $h_{11}S_2^1G_4^2S_4^4G_2^4S_1^2g_{11}p^1$. This expression results from two applications of the propagation product: $w_1 = h_{11}S_1^\mu \left(G_\mu^\nu S_\nu^\mu\right) \left(G_\mu^\nu S_\nu^\mu\right) g_{\mu a}p^a$. To obtain the wave expression at the step N, we make:

$$w_1 = h_{11}S_1^\mu \left(G_\mu^\nu S_\nu^\mu\right)^N g_{\mu a}p^a \quad (7)$$

The last equation is a general one to obtain the wave at any port of the network. The equation always starts with some product $g_{\mu a}p^a$ and ends with the product $h_{aa}S_a^\mu$. Between these terms, the product $\left(G_\mu^\nu S_\nu^\mu\right)$ is applied as any time as the process is running far.

4 Extraction of the wave exact expression

Once the previous process is applied, it is possible to extract from the wave vector an expression of a particular intensity on one port. It was the case of the expression $h_{11}S_2^1G_4^2S_4^4G_2^4S_1^2g_{11}p^1$. Now, we want to obtain from this symbolique expression, the real one of the wave. We know that some fields can decrease in a $1/R$ law, where R is the distance treveled. And, if the propagation matrix is made of function like $1/R$; we cannot retrieve the $1/(R_1 + R_2)$ expressions from the products $G_4^2G_2^4$ for example. Inspired by the γ -matrices algebra describes in the Feynman's course on quantum electrodynamics[4], we find the idea to replace each of the previous element by γ -matrix like. Each S_j^i element is replaced by a matrix of the form:

$$S_j^i = \begin{bmatrix} s(i,j) & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

where $s(i,j)$ is a matrix given by:

$$s(i,j) = \begin{bmatrix} (s_{ij})_1 & 0 \\ 0 & (s_{ij})_2 \end{bmatrix} \quad (9)$$

Both element $(s_{ij})_k$ are the S parameter value between the port i and j for a given polarisation k . In our example we consider only one polarisation. 1 is the 2x2 identity matrix and 0 a zero 2x2 matrix. The G_j^i element are replaced by:

$$G_j^i = \begin{bmatrix} 1 & 0 \\ 0 & x_{ij} \end{bmatrix} \quad (10)$$

with 1 and 0 matrix the same 2x2 matrices are previously and x_{ij} being the matrix:

$$x_{ij} = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \quad (11)$$

x is the distance link with the propagation element G_j^i .
The source vector must be defined in the form:

$$p^1 = \begin{bmatrix} (p_0)_1 \\ (p_0)_2 \\ 0 \\ 1 \end{bmatrix} \quad (12)$$

Here, $(p_0)_k$ is the source for the polarisation k . Element like h_{jj} or g_{ii} are 4x4 identity matrix with their coefficients (j, j) or (i, i) equals to the transformation coefficient.

We obtain a similar process as for the global network, but the matrices have their dimensions fixed (4x4) and the $G_j^i S_k^j$ products can be synthesized in γ -matrix given by:

$$\gamma_k^i = \begin{bmatrix} (s_{ij})_1 & 0 & 0 & 0 \\ 0 & (s_{ij})_2 & 0 & 0 \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s(j, k) & 0 \\ 0 & x_{ij} \end{bmatrix} \quad (13)$$

Using these definitions, the component of the wave vector $h_{11} S_1^2 G_2^4 S_4^4 G_4^2 S_2^1 g_{11} p^1$ becomes:

$$\tilde{w}_1 = h_{11} S_1^2 \gamma_2^4 \gamma_4^1 g_{11} p^1 \quad (14)$$

Applying the previous rules we obtain:

$$\tilde{w}_1 = \begin{bmatrix} h_{11} S_1^2 \gamma_2^4 \gamma_4^1 g_{11} p^1 \\ 0 \\ x + x \\ 1 \end{bmatrix} \quad (15)$$

5 Use of the result

The 4-vector $\tilde{\omega}_1$ allows to deduce the expression of the waves modeled. If we consider fields depending on the green function under a Lorentz's gauge, we write:

$$\hat{w}(1) = \frac{|(\tilde{w}_1)^1|}{4\pi(\tilde{w}_1)^3} e^{-ik(\tilde{w}_1)^3} \quad (16)$$

If the waves concerned are power ones at wavelength λ , the expression becomes:

$$\hat{w} = |(\tilde{w}_1)^1| \frac{\lambda^2}{16\pi^2 [(\tilde{w}_1)^3]^2} \quad (17)$$

For line modeling with a propagation constant β :

$$\hat{w}(1) = |(\tilde{w}_1)^1| e^{-i\beta(\tilde{w}_1)^3} \quad (18)$$

or for primary plane wave fields (electric or magnetic far fields):

$$\hat{w}(1) = \frac{|(\tilde{w}_1)^1|}{4\pi [(\tilde{w}_1)^3]^2} e^{-ik(\tilde{w}_1)^3} \quad (19)$$

etc. Depending on the physical model which is backward the graph representation, the amplitudes and distances traveled available as element 1, 2 and 3 of the " γ -wave vector" on port k : \tilde{w}_k , give all the freedom to compute the wave function wanted.

6 Composite waves

If many ways are explored after a high order of computation of the product GS , many terms can appear on a port. In this case, the previous operations give as results various third term of distances. One will be $x + y + z + \dots$ and a second $2x + r + \dots$ if the tube distances are x, y, z, \dots, r, \dots . As a consequence, the various terms cannot be confused due to the fact that reported as exponents in the Green's functions, they will be linked with different exponent phases. For example, in the case presented figure 1, the γ -wave vector is:

$$\tilde{w}_1 = \begin{bmatrix} h_{11}S_1^2\gamma_2^4\gamma_4^1g_{11}p_0 + h_{11}S_1^3\gamma_3^7\gamma_7^1g_{11}p_0 \\ 0 \\ 2x_{24} + 2x_{37} \\ 1 \end{bmatrix} \quad (20)$$

The first operation in presence of wave addition in the γ -wave vector is to separate the component with respect to their order of apparition. We write:

$$\tilde{w}_1 = \begin{bmatrix} h_{11}S_1^2\gamma_2^4\gamma_4^1g_{11}p_0 \\ 0 \\ 2x_{24} \\ 1 \end{bmatrix} + \begin{bmatrix} h_{11}S_1^3\gamma_3^7\gamma_7^1g_{11}p_0 \\ 0 \\ 2x_{37} \\ 1 \end{bmatrix} \quad (21)$$

Under this form, the various waves involved at this computation step (which is the second one) becomes separated. We see that the computation step is independant from the real time steps. The exponents in the wave delays can be different. We obtain here in the Lorentz's gauge representation:

$$\tilde{w}_1 = \frac{h_{11}}{4\pi} \left\{ S_1^2 |\gamma_2^4\gamma_4^1| \frac{e^{-ik2x_{24}}}{2x_{24}} + S_1^3 |\gamma_3^7\gamma_7^1| \frac{e^{-ik2x_{37}}}{2x_{37}} \right\} g_{11}p_0 \quad (22)$$

7 A simple example

We consider the basic system where two junctions and four ports are involved. It represents two antennas in communication. The system being matched, its G and S matrices are defined following:

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & G_3^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ S_2^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & S_4^3 & 0 \end{bmatrix} \quad (23)$$

We define too the g and h matrix:

$$g = \begin{bmatrix} g_{11} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad h = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & h_{44} \end{bmatrix} \quad (24)$$

and finally the source vector:

$$p^1 = \begin{bmatrix} p_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

Making the matricial product: $hSGSp^1$ we find the vector result:

$$\omega_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ h_{44}S_4^3G_3^2S_2^1g_{11}p_1 \end{bmatrix} \quad (26)$$

If we apply one more time G before to apply h , ce vector result still equal to zero whatever the product GS is then applied. This is because, the system being matched, no more wave events appears. Now the next step consists in the exploitation of the result vector ω_4 . To do so, we replace each element by its equivalent γ -matrix. It means (remember that here, the matrix dimension is always 4x4, whatever the port space dimension. The fact that here the dimensions are the same is purely an hazard.):

$$h_{44} = \begin{bmatrix} h_{44} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad g_{11} = \begin{bmatrix} g_{11} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

$$p_1 = \begin{bmatrix} p_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (28)$$

$$S_2^1 = \begin{bmatrix} S_2^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S_4^3 = \begin{bmatrix} S_4^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

If D is the distance between the emitter and the receiver:

$$G_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & D \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (30)$$

Starting from these definitions we can construct the γ_3^1 propagation matrix:

$$\gamma_3^1 = G_3^2 S_2^1 = \begin{bmatrix} S_2^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & D \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

Now we compute $h_{44} S_4^3 \gamma_3^1 g_{11} p_1$ to find:

$$\tilde{\omega}_4 = \begin{bmatrix} h_{44} S_4^3 S_2^1 g_{11} p_1 \\ 0 \\ D \\ 1 \end{bmatrix} \quad (32)$$

from which we deduce:

$$\hat{\omega}_4 = (\tilde{\omega})^1 \left(\frac{\lambda}{4\pi(\tilde{\omega})^3} \right)^2 = h_{44} S_4^3 S_2^1 g_{11} p_1 \left(\frac{\lambda}{4\pi D} \right)^2 \quad (33)$$

With: $g_{11} = G1$ gain of the first antenna, $h_{44} = G2$, gain of the second antenna with $S_4^3 = S_2^1 = 1$ as the system is matched. The result becomes: $G1G2p_1 \left(\frac{\lambda}{4\pi D} \right)^2$ which is the Friis's formula for the transmission losses between two antennas separated by a distance D .

8 Conclusion

We have described a technique to reproduce the wave process whatever the wave type involved. Firstly, the technique can be very useful to explain the traveling wave process in general. Secondly, it gives a generic method to resolve various problems of wave distribution without any inverse operation as in integral method or fluence graph approach[5]. Using junctions with multiple tube connections, it is possible to follow with a high accuracy any kind of topology. The computation time still constant, if the result is memorized at each step. In this case, any step more consists in a simple multiplication by the propagation product GS .

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